>>> EdDSA: Not just ECDSA with a twist

>>> Math 445 (Introduction to Cryptography)

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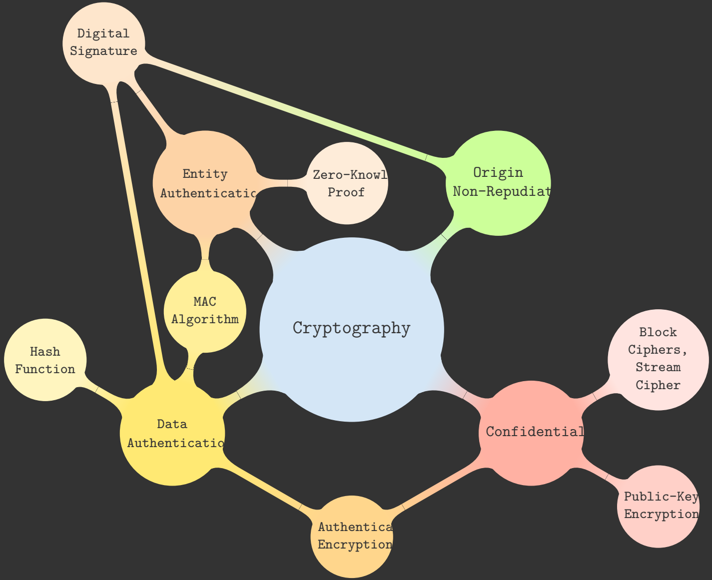

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   * https://www.keylength.com/en/compare/

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Choose a cryptographic hash function H with appropriate domain and codomain. The key generation algorithm outputs a pair (k, Q) such that $Q = [k]P \in \mathbb{G} = \langle P \rangle$ with $|\mathbb{G}| = \ell \nmid p$, where k is the *secret signing key* and Q is the *public verification key*.

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Signing $(\mathbb{G}, P, k, \mathsf{H}, m)$

1. $t \xleftarrow{\$} \{1, \ldots, \ell - 1\}$
2. $R \leftarrow [t]P$
3. $r \leftarrow x(R) \pmod{\ell}$
4. if $r = 0$ then goto Step 1.
5. $e \leftarrow \mathsf{H}(m)$
6. $s \leftarrow (e + rk)t^{-1} \pmod{\ell}$
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Verification $(\mathbb{G}, P, Q, \mathsf{H}, m, \sigma)$

1. $e \leftarrow \mathsf{H}(m)$
2. $u_1 \leftarrow es^{-1} \pmod{\ell}$, $u_2 \leftarrow rs^{-1} \pmod{\ell}$
3. $T \leftarrow [u_1]P + [u_2]Q$
4. return $r \stackrel{?}{=} x(T) \pmod{\ell}$

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2. ECDSA was invented only to circumvent patents in Schnorr signatures. Unfortunately, ECDSA does not come with a proof of security, while Schnorr signatures did.

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2. NSA: https://en.wikipedia.org/wiki/Dual_EC_DRBG
   * At the CRYPTO 2007 conference rump session, Dan Shumow and Niels Ferguson presented a potential backdoor in the NIST/NSA specified Dual_EC_DRBG cryptographically secure pseudorandom number generator. The backdoor was confirmed to be real in 2013 as part of the Edward Snowden leaks.

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   * Birationally equivalent to Montgomery form.
   * If a is a square and d is a non-square, then a single addition formula works for all possible inputs.

>>> Safe curves

The Dual_EC_DRBG algorithm was based on P-256 curve

$$y^2 = x^3 - 3x +$$

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$\mathcal{P}$ ---- $\mathbb{G}, p, Q$ ---- $\mathcal{V}$

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$\xrightarrow{\text{commitment: } R}$

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   * Data authentication: Assurance of the integrity of data.

>>> Further reading

- Hüseyin Hışıl, 2010, *Elliptic Curves, Group Law, and Efficient Computation.* §1.1 and 2.3.4
  https://eprints.qut.edu.au/33233/
- Nigel P. Smart, 2016, *Cryptography Made Simple.* §21.3
- Simon Josefsson and Ilari Liusvaara, 2017, *Edwards-Curve Digital Signature Algorithm.* §1
  https://www.rfc-editor.org/info/rfc8032
- Steven D. Galbraith, 2018, *Mathematics of Public Key Cryptography.* §9.12
  https://www.math.auckland.ac.nz/~sgal018/crypto-book/crypto-book.html
- David Wong, 2021, *Real World Cryptography.* §7.3.4
- Luís T. A. N. Brandão and Michael Davidson, 2022, *Notes on Threshold EdDSA/Schnorr Signatures.* Figures 1 and 2
  https://csrc.nist.gov/pubs/ir/8214/b/ipd