Beautiful Repetitions

5-minute introduction to Iterations & Fractals

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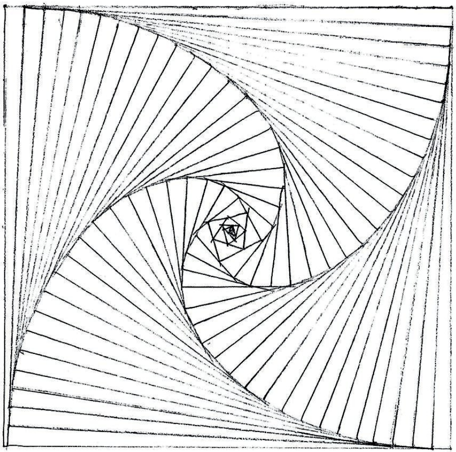
National Science Day

Is this ugly?



Technical Terms - I

Iteration OR Iterative Process

Any action which is performed repeatedly will be referred to as an *iteration*. The specific rule used to do the iteration is the *iterative function*. The input value with which we start the iteration is the *seed* of the iteration.

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Iteration Sequence OR Orbit OR Trajectory

The sequence which is generated by an iterative function together with a seed is an iteration sequence. A convenient short form for this is: $< x; f >$, where x is seed and f is *iterative function.*

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Cycle

An orbit that returns to its starting value after a certain number of steps is called a *cycle*. The number of entries in a cycle is called its *length*.

Algebraic Iterations

The $3n + 1$ conjecture concerns the following innocent seeming arithmetic procedure applied to integers described by *iterative function*, given by :

$$f(n) = \begin{cases} \frac{3n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

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Statement of Conjecture

Starting from any positive integer n, iterations of the function $f(n)$ will eventually reach the number 1. Thereafter iterations will cycle, taking successive values $1, 2, 1, \ldots$

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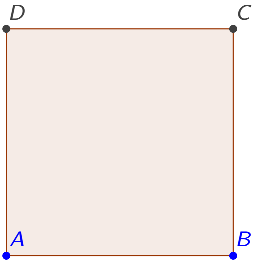
Can you Prove it?

This is around 85 year old conjecture. This conjecture was independently discovered by many mathematicians so is also called *Collatz conjecture*, *Ulam conjecture*, *Kakutanis problem*, *Thwaites conjecture*, *Hasses algorithm* and *Syracuse problem*.

Geometric Iterations

Let $\mathcal{S}_0$ be a square, and let its vertices be labelled in cyclic order as A,B,C,D.



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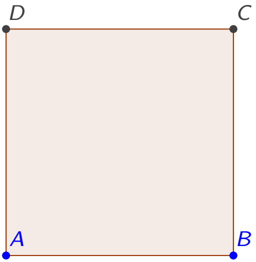


Let k be a number lying between 0 and 1.

Geometric Iterations

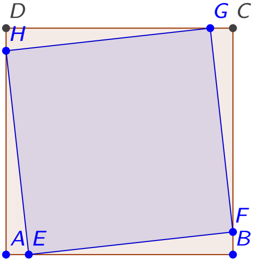
Let points E,F,G,H, respectively, be located on the sides AB, BC, CD, DA of $\mathcal{S}_0$, according to the following rule:

$$\frac{AE}{AB} = \frac{BF}{BC} = \frac{CG}{CD} = \frac{DH}{DA} = k$$

The segments EF, FG, GH, HE are drawn, giving another square, $\mathcal{S}_1$, which we take to be the output of a function with input $\mathcal{S}_0$.

Geometric Iterations

For $k = 0.1$ we will get following $\mathcal{S}_1$:



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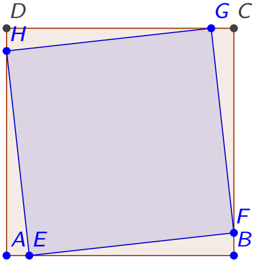

Observe that $\mathcal{S}_1$ is smaller than $\mathcal{S}_0$ and is wholly contained within it; it has same center as its parent square, but has been turned through a small angle.

Geometric Iterations

Now what we defined above was a process which we will iterate again and again.
If we iterate the process defined above, then after 20 iterations we will get:

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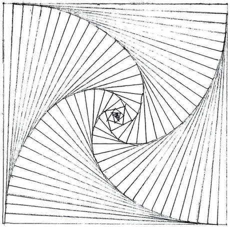

[Source: gldfsh.deviantart.com]

Voilà!

We are surrounded by Fractals

[Source: Hudhud Cyclone, ABP Live]

The spiral is an extremely common fractal in nature, found over a huge range of scales.

Technical Terms - II

Fractal

A fractal is a never ending pattern that repeats itself at different scales. A fractal is made by *iterations*. **Eg:** *Cauliflower; coastline*

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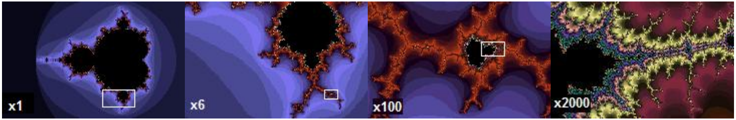
Self-Similarity

The property of fractal to repeat itself at different scales is called *self-similarity*.This is analogous to zooming in on digital images to uncover finer, previously invisible, new structure. If this is done on fractals, however, no new detail appears; the same pattern repeats over and over.

Eg: Mandelbrot set fine detail resembles the detail at low magnification.



Algebraic Fractal

We can create fractals by iterating a simple equation. Since the equations must be calculated thousands of times, we need computers to study them.

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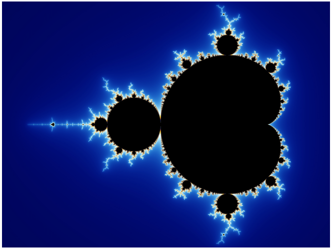
Mendelbrot Set



©Wolfgang Beyer [Wikipedia]

This is a false colour image of a set, colour coded according to behaviour of elements of the set towards an iterative function.

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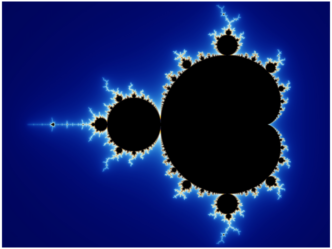

©Wolfgang Beyer [Wikipedia]

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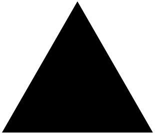
I can't explain the Mathematics behind this fractal since it involves iterations on set of complex numbers ($\mathbb{C}$) !

Geometric Fractal

1. Start with a solid equilateral triangle



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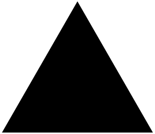
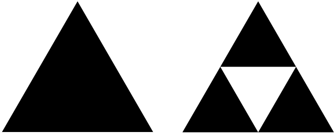


2. Divide it into four equal pieces, and remove the middle piece.



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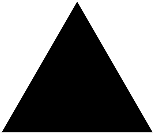

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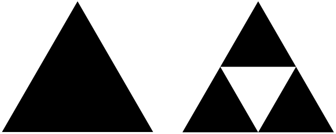
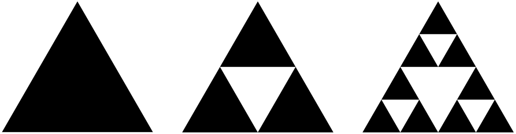
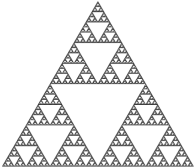


3. Then divide each of the three remaining triangles into four equal pieces, and remove the middle piece of each one.



Geometric Fractal

If this pattern is continued forever, then the result is *Sierpinski's triangle*.



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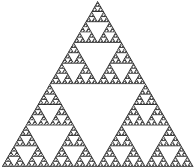
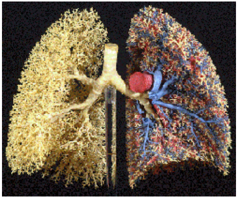


If you made three copies of the triangle, and shrunk them down by a factor of one-half, then you could cover the original triangle. Because the figure has this self-similarity property, it is a *fractal*

Fractals in Everyday life

- The bronchial network in human respiratory system is a fractal



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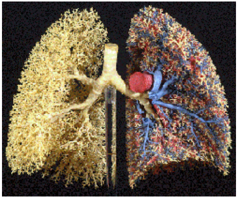

©E. Weibel

- The artificial landscapes like *Mountains*, etc. which you see in cinema are actually fractals. *(they just "appear" to be mountains)*

[Source: djconnel.com/Vue]

This is just start....

Fractals constitute a relatively modern discovery; they date to the latter half of the 20th century and may be said to have originated in the work of the French Mathematician *Benoit Mandelbrot*. So, in this presentation I have shown you just tip of iceberg. Go explore yourself . . .

Video

- Fractals and the art of Roughness by Benoit Mandelbrot; *TED Talk*

Books

- A Gateway to Modern Mathematics : Adventures in Iterations I & II by Shailesh A Shirali; *Ramanujan Mathematical Society : Little Mathematical Treasures, ISBN: 9788173716263 & 9788173716928*
- Chaos, Fractals and Self-Organisation by Arvind Kumar; *National Book Trust, ISBN: 9788123715964*

Thank you !

Curious?

For any further discussion on mathematics feel free to write me at:

gaurishkorpal01@gmail.com

Download

This presentation can be downloaded from:

gaurish4math.wordpress.com/downloads/my-notes/