Geometry Around Us

An Introduction to Non-Euclidean Geometry

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Science Day Celebration

Morley's Miracle

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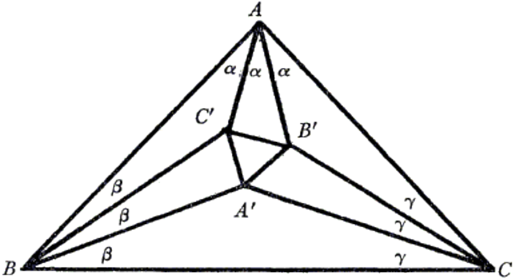
Theorem

In any triangle, trisector lines intersect at three points, that are vertices of an equilateral triangle.



A. Bogomolny, Morley's Theorem: Proof by R. J. Webster, Interactive Mathematics Miscellany and Puzzle, `http://www.cut-the-knot.org/triangle/Morley/Webster.shtml`

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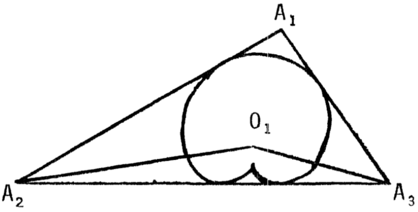

By F. Morley, On the Intersections of the Trisectors of the Angles of a Triangle, Journal of the Mathematical Association of Japan for Secondary Education, 6 (December 1924), 260-262. [taken from pp. 275, EUREKA Vol. 3, No. 10, 1977]

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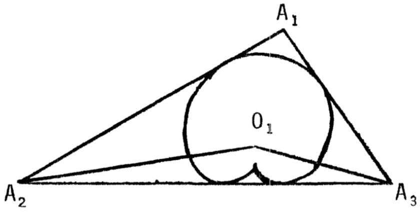

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But. . .

Morley's theorem does not hold in non-eucledian geometry (we will soon see what are they!).

Euclid's Postulates

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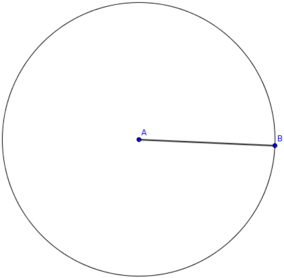
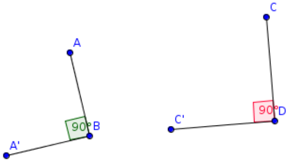
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center



4 All right angles are congruent.



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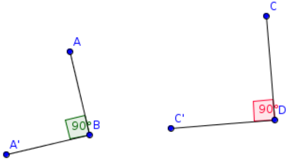
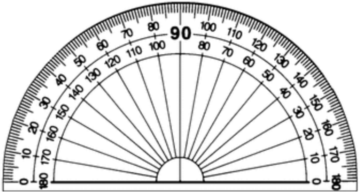
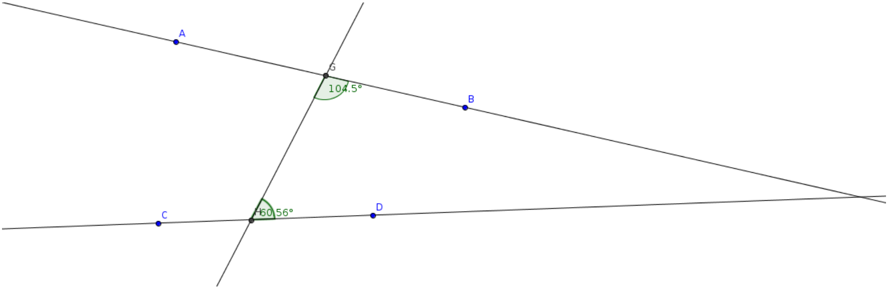


We measure angles using an instrument called *Protractor*



5 If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.



Here the sum of marked angles is $104.5^\circ + 60.56^\circ = 165.06^\circ < 180^\circ = 2 \times 90^\circ$

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Triangle gets distorted...

A consequence of the parallel lines postulate in Euclidean geometry implies that the interior angles of a triangle always add up to 180 degrees. Thus modifying the parallel postulate modifies the possibilities for the sum of the interior angles of a triangle.

Let S be the sum of the interior angles of a triangle in a given geometry, then

- Hyperbolic geometry: $S < 180$

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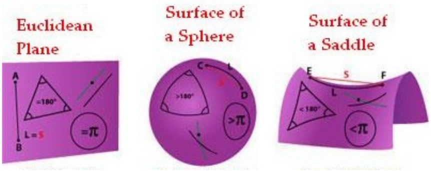

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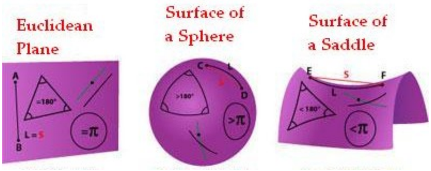

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The Fathers...

Around 1830, János Bolyai (Hungary) and Nikolai Ivanovich Lobachevsky (Russia) separately published treatises on hyperbolic geometry. So both mathematicians, independent of each other, are the basic authors of non-Euclidean geometry.

Hyperbolic Geometry

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By Toby Hudson [CC BY-SA 3.0 or GFDL], via Wikimedia Commons; https://commons.wikimedia.org/wiki/File%3AFolded_Coral_Flynn_Reef.jpg

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Rectangles don't exist in hyperbolic geometry. Moreover there are infinitely many parallels to l through a given point P. If two triangles are similar, they are congruent.

Hyperbolic Geometry Around Us

Hyperbolic plane geometry is the geometry of some special saddle surfaces. An example of two-dimensional saddle surface is the hyperbolic paraboloid [$z = x^2 - y^2$] which looks like actual horse saddle

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By BLW [Public domain], via Wikimedia Commons; https://commons.wikimedia.org/wiki/File%3ACollegiateEventer.jpg

Because a *hyperbolic paraboloid* has two distinct lines that lie on the surface through every one of its points (doubly ruled), it can be used to construct a saddle roof from straight beams.

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An example of that is the Ochota railway station in Warszawa, Poland.

By Panek [GFDL or CC BY-SA 4.0-3.0-2.5-2.0-1.0], via Wikimedia Commons;
`https://commons.wikimedia.org/wiki/File%3AW-wa_Ochota_PKP-WKD.jpg`

Spherical Geometry

It is the study of figures on the surface of a sphere, as opposed to the type of geometry studied in plane geometry or solid geometry. There are also no parallel lines and the straight lines are great circles, so any two lines meet in two points.

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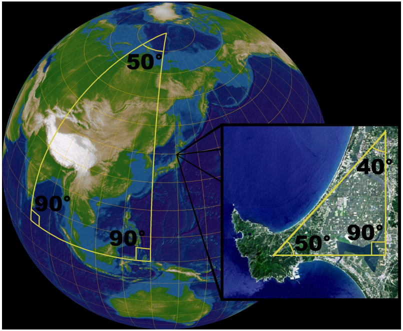

By Lars H. Rohwedder, Sarregouset [GFDL or CC BY-SA 3.0], via Wikimedia Commons, https://commons.wikimedia.org/wiki/File%3ATriangles_(spherical_geometry).jpg

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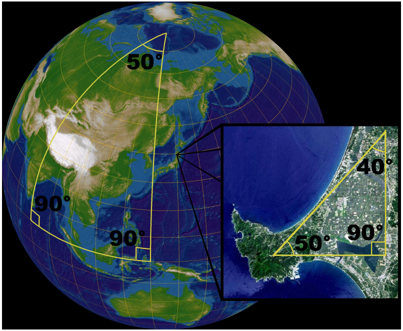

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A spherical triangle is defined by its three angles. There is no concept of similar triangles in spherical geometry.

Spherical Geometry Around Us

Great-circle navigation is the practice of navigating a vessel (a ship or aircraft) along a great circle. A great circle track is the shortest distance between two points on the surface of a sphere.

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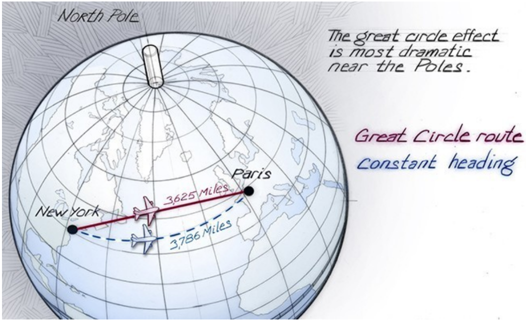

By Bruce Morser, National Air and Space Museum, Smithsonian Institution (2012), `https://timeandnavigation.si.edu/multimedia-asset/great-circle-route`

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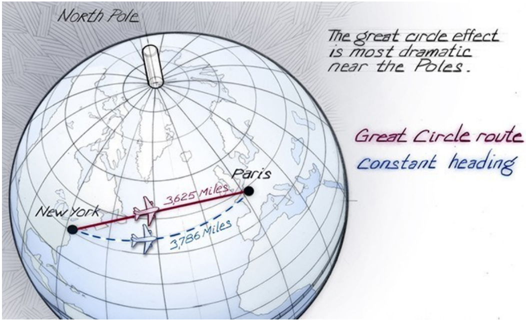

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If the above globe is flattened out on a flat surface, the straight red line would appear curved and the curved dashed blue line would appear straight.

What's Next?

The discovery of non-Euclidean geometries posed an extremely complicated problem to physics, that of explaining whether real physical space was Euclidean as had earlier been believed, and, if it is not, to what what type of non-Euclidean spaces it belonged. This problem is still not completely resolved.

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- NonEuclid by Joel Castellanos, Joe Dan Austin & Ervan Darnell; *Interactive Javascript Software for Creating Straightedge and Collapsible Compass Constructions in the Poincaré Disk Model of Hyperbolic Geometry.* `http://www.cs.unm.edu/~joel/NonEuclid/NonEuclid.html`

Thank you !

Curious?

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